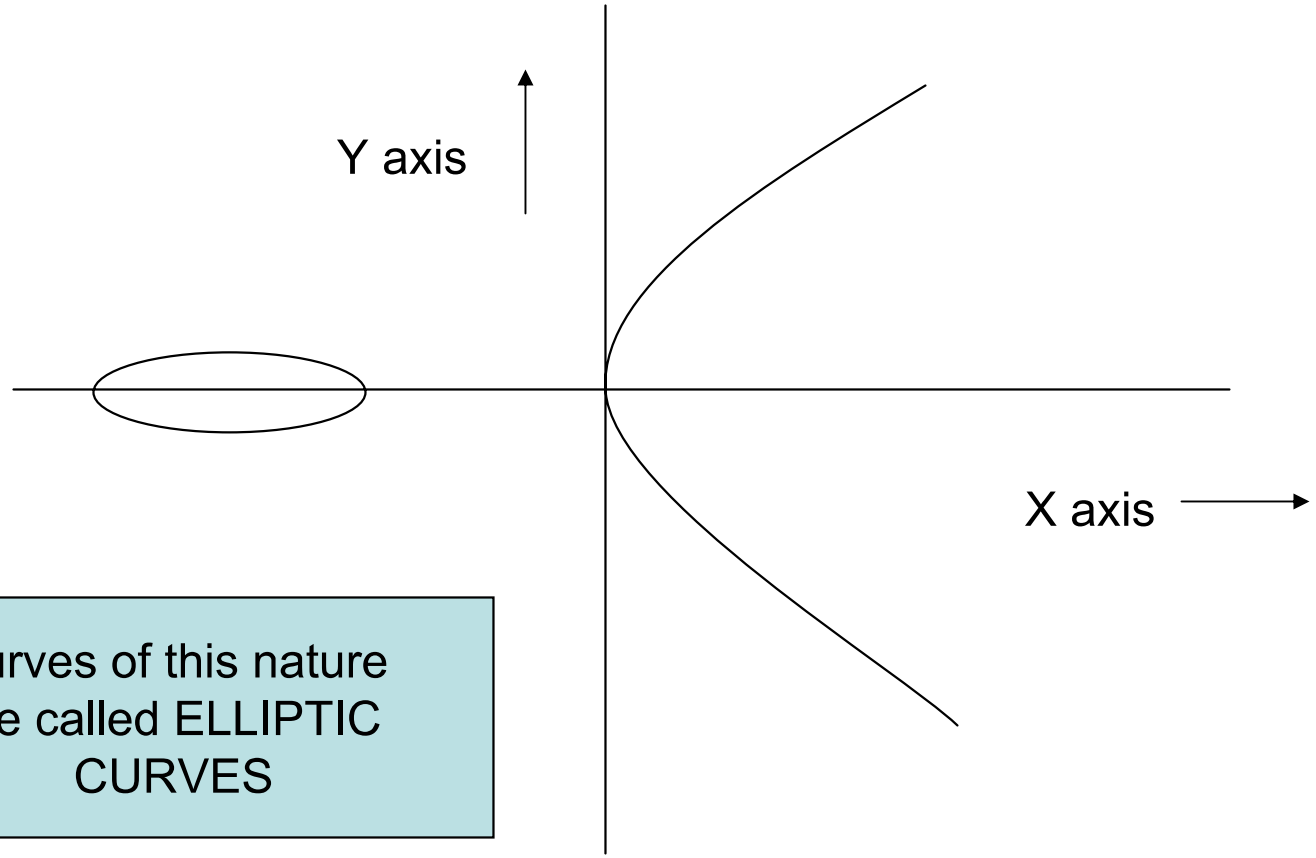
Graphical Representation



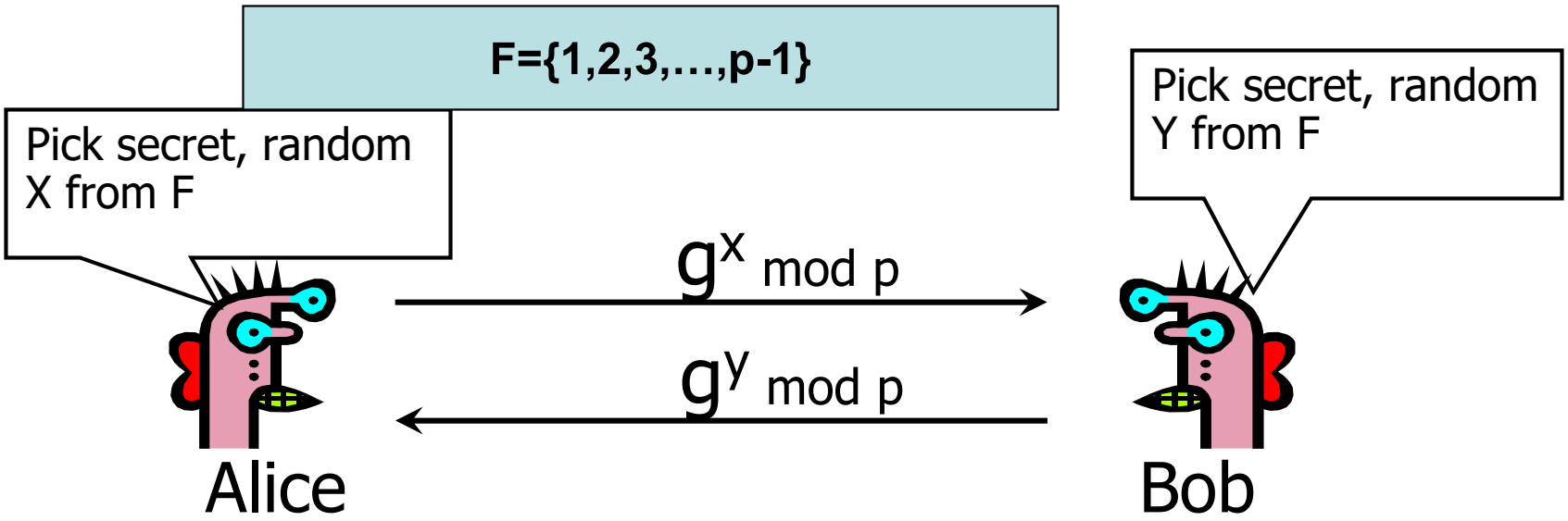
Curves of this nature are called ELLIPTIC CURVES

Elliptic curves in Cryptography

- Elliptic Curve (EC) systems as applied to cryptography were first proposed in 1985 independently by Neal Koblitz and Victor Miller.

- The discrete logarithm problem on elliptic curve groups is believed to be more difficult than the corresponding problem in (the multiplicative group of nonzero elements of) the underlying finite field.

Discrete Logarithms in Finite Fields



Compute $k=(g^y)^x=g^{xy}$ mod p

Compute $k=(g^x)^y=g^{xy}$ mod p

Eve has to compute g^{xy} from g^x and g^y without knowing x and y…
She faces the Discrete Logarithm Problem in finite fields

Elliptic Curve on a finite set of Integers

- Consider $y^2 = x^3 + 2x + 3 \pmod 5$

  $x = 0 \Rightarrow y^2 = 3 \Rightarrow$ no solution $\pmod 5$

  $x = 1 \Rightarrow y^2 = 6 = 1 \Rightarrow y = 1,4 \pmod 5$

  $x = 2 \Rightarrow y^2 = 15 = 0 \Rightarrow y = 0 \pmod 5$

  $x = 3 \Rightarrow y^2 = 36 = 1 \Rightarrow y = 1,4 \pmod 5$

  $x = 4 \Rightarrow y^2 = 75 = 0 \Rightarrow y = 0 \pmod 5$

- Then points on the elliptic curve are

  $(1,1)\ (1,4)\ (2,0)\ (3,1)\ (3,4)\ (4,0)$
  and the point at infinity: ∞

Using the finite fields we can form an Elliptic Curve Group where we also have a DLP problem which is harder to solve…

Definition of Elliptic curves

- An elliptic curve over a field ***K*** is a nonsingular cubic curve in two variables, *f(x,y) =0* with a rational point (which may be a point at infinity).

- The field ***K*** is usually taken to be the complex numbers, reals, rationals, algebraic extensions of rationals, p-adic numbers, or a **finite field**.

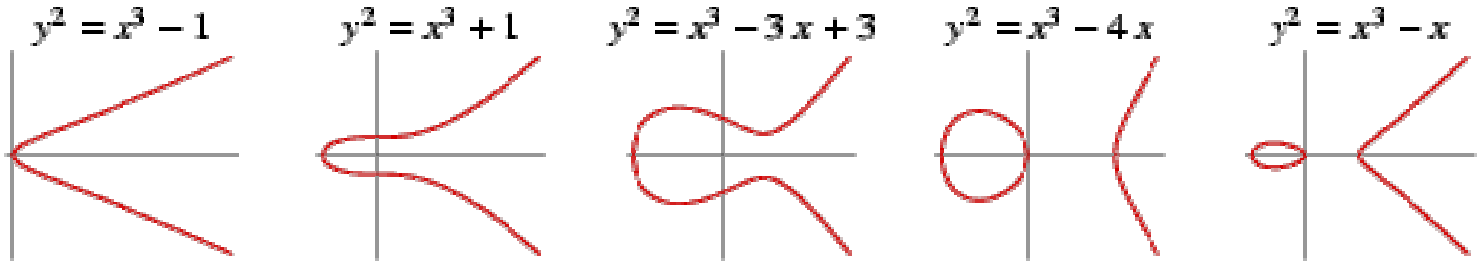
- Elliptic curves groups for cryptography are examined with the underlying fields of $\boldsymbol{F_p}$ (*where p>3 is a prime*) and $\boldsymbol{F_2^m}$ (*a binary representation with* 2^m *elements*).

General form of a EC

- An *elliptic curve* is a plane curve defined by an equation of the form

$$y^2 = x^3 + ax + b$$

Examples



The Abelian Group

Given two points P,Q in *E(Fp)*, there is a third point, denoted by *P*+*Q* on *E(Fp)*, and the following relations hold for all P,Q,R in *E(Fp)*

- $P + Q = Q + P$ (*commutativity*)
- $(P + Q) + R = P + (Q + R)$ (*associativity*)
- $P + O = O + P = P$ (*existence of an identity element*)
- there exists $(-P)$ such that $-P + P = P + (-P) = O$ (*existence of inverses*)

What Is Elliptic Curve Cryptography (ECC)?

- Elliptic curve cryptography [ECC] is a **public-key** cryptosystem just like RSA, Rabin, and El Gamal.
- Every user has a **public** and a **private** key.
  - Public key is used for encryption/signature verification.
  - Private key is used for decryption/signature generation.
- Elliptic curves are used as an extension to other current cryptosystems.
  - Elliptic Curve Diffie-Hellman Key Exchange
  - Elliptic Curve Digital Signature Algorithm

Using Elliptic Curves In Cryptography

- The central part of any cryptosystem involving elliptic curves is the **elliptic group**.
- All public-key cryptosystems have some underlying mathematical operation.
  - RSA has exponentiation (raising the message or ciphertext to the public or private values)
  - ECC has point multiplication (repeated addition of two points).

Generic Procedures of ECC

- Both parties agree to some publicly-known data items
  - The **elliptic curve equation**
    - values of ***a*** and ***b***
    - prime, ***p***
  - The **elliptic group** computed from the elliptic curve equation
  - A **base point**, B, taken from the elliptic group
    - Similar to the generator used in current cryptosystems
- Each user generates their public/private key pair
  - Private Key = an integer, x, selected from the interval [1, p-1]
  - Public Key = product, Q, of private key and base point
    - (Q = x*B)

Example – Elliptic Curve Cryptosystem Analog to El Gamal

- Suppose Alice wants to send to Bob an encrypted message.
  - Both agree on a base point, B.
  - Alice and Bob create public/private keys.
    - Alice
      - Private Key = a
      - Public Key = $P_A = a * B$
    - Bob
      - Private Key = b
      - Public Key = $P_B = b * B$
  - Alice takes plaintext message, M, and encodes it onto a point, P_M, from the elliptic group

Why use ECC?

- How do we analyze Cryptosystems?
  - How difficult is the underlying problem that it is based upon
    - RSA – Integer Factorization
    - DH – Discrete Logarithms
    - ECC - Elliptic Curve Discrete Logarithm problem
  - How do we measure difficulty?
    - We examine the algorithms used to solve these problems